

Frost Depth and Distribution From a Heat Flow Model^{1/}

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ABSTRACT

A soil heat flow model has been developed and programmed to predict the depth, distribution, and duration of frost in northern watersheds. The model assumes that heat loss from the soil surface (Q_F) is a function of the thermal conductivity (K_F) and temperature gradient ($\frac{dv}{dx}$) across a frozen surface layer. Surface heat loss at any given time must be balanced by heat flow from the unfrozen soil below. Unfrozen soil heat flow equals the sum of heat transferred by means of the soils thermal conductivity (K_S) and temperature gradient ($\frac{dv}{dx}$), the latent heat (L) released in freezing transmitted water ($K_w \frac{d\psi}{dx}$), and the change in soil heat content ($C_S \Delta V$). The operation of the model and predicted frost depth and distribution are presented for various conditions of snow depth, elevation, and soil water content.

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Soil factors that influence frost depth and the relationship between predicted frost depth and watershed behavior are discussed.

INTRODUCTION

The time and magnitude of stream response to hydrologic events is controlled by the path water takes in moving over or through the soils of a watershed to a stream. The availability of water and the path it follows in northern watersheds is modified by the presence or absence of frost. Thus, the ability to predict the presence, type, and distribution of frost is essential to a full modeling of hydrologic behavior of northern watersheds during winter snow accumulation and spring melt periods.

Frost development and distribution followed by subsequent thawing is a complex process. At any point, rate of frost penetration, type of frost, and rate of thaw are functions of surface temperature, soil water content, soil texture, soil structure, soil color, and type and thickness of soil cover (1, 2, 3, 5, 6, 7, 8, 9). Some of these parameters have been used as criteria for determining the frost susceptibility of a soil (4, 6). However, since soil freezing and thawing are really a heat flow problem (4), the stated parameters are important only to the extent that they modify or control the mechanisms of heat flow in the soil.

The manuscript presents the initial phases of the development of a frost depth and distribution model, based on soil heat flow characteristics. The model uses average daily air temperatures and the thermal conductivity properties of frozen and unfrozen soil to produce a constantly changing evaluation of soil freezing or thawing. The development of the model, the soil factors considered, and the results predicted by the model are discussed.

SOIL FROST MODEL

The model assumes heat flow (Q_F) through soil frost can be computed by the equation

$$Q_F = K_F \frac{dv}{dx} \quad (1)$$

where

Q_F = heat flux ($\text{cal}/\text{cm}^2 \text{ sec}$)

K_F = thermal conductivity of frozen soil ($\text{cal}/\text{cm sec}^\circ\text{C}$)

v = temperature ($^\circ\text{C}$)

x = distance (cm)

This means dv equals the difference between average daily air temperature and the temperature at the bottom of the frost layer (which is assumed to be 0°C). Then, dx equals the thickness of the frost layer.

For a given time interval, Q_F must be balanced by heat removal (Q_v) from the unfrozen zone below the frozen soil. Q_u is defined as the sum of heat transferred by the thermal conductivity properties of the soil matrix, the latent heat of fusion associated with water migration, and losses in heat content of the soil. That is

$$Q_u = k_s \frac{dv}{dx} + LK_w \frac{d\psi}{dx} + C_s \Delta v \quad (2)$$

where

k_s = thermal conductivity of the unfrozen soil (cal/cm sec $^{\circ}$ C)

v & x = as above

L = latent heat of fusion

K_w = capillary conductivity of soil (cm/sec)

C_s = heat capacity of the soil (cal/ $^{\circ}$ C)

Δv = change in soil temperature ($^{\circ}$ C)

ψ = capillary potential

Here $\frac{dv}{dx}$ and $\frac{d\psi}{dx}$ are the gradients of soil temperature and water tension found just below the 0° C isotherm.

Equation (2) assumes that thermal conductivity and water migration are independent modes of heat transfer in the soil. However, these two modes of heat transfer are closely related and are treated separately as a numerical and practical approximation.

The complete model, depicted in Figure 1 for soil with no snow cover, states that $Q_F = Q_u$ over a given time, or in

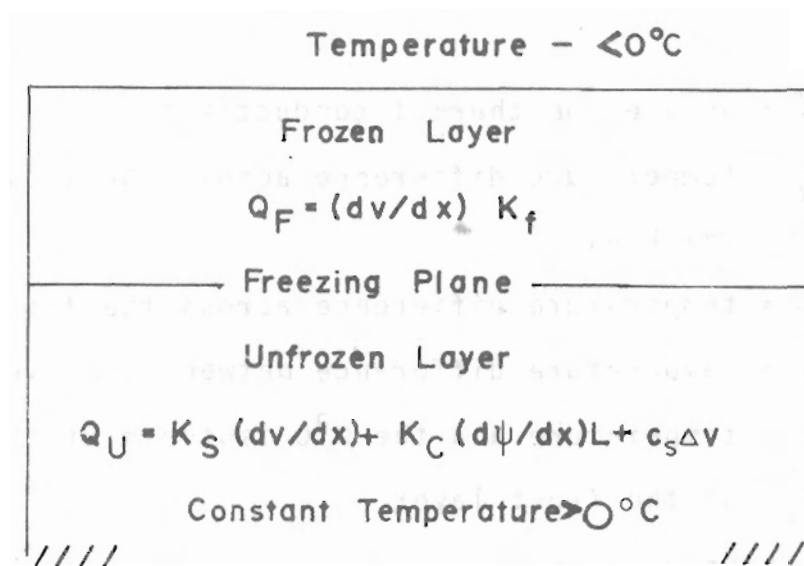


Figure 1. Schematic diagram of physical condition assumed in frost model.

terms of equations (1) and (2) states

$$K_F \frac{dv}{dx} = K_S \frac{dv}{dx} + LK_W \frac{d\psi}{dx} + C_S \Delta v \quad (3)$$

With snow cover, heat flow from the 0°C isotherm to the air-solid interface is a function of some average thermal conductivity, \bar{K} , for frozen soil and snow.

That is,

$$Q_F = \bar{K} \frac{d(v_{\text{snow}} + v_f)}{d(x_{\text{snow}} + x_f)}$$

where

\bar{K} = an average thermal conductivity

v_{snow} = temperature difference across the snow thickness

v_f = temperature difference across the frost layer

$d(v_s + v_f)$ = temperature difference between the average air temperature and the 0°C isotherm at the bottom of the frost layer

x_{snow} = snow depth

x_f = frost depth

Under these conditions a weighted average (\bar{K}) can be shown to equal

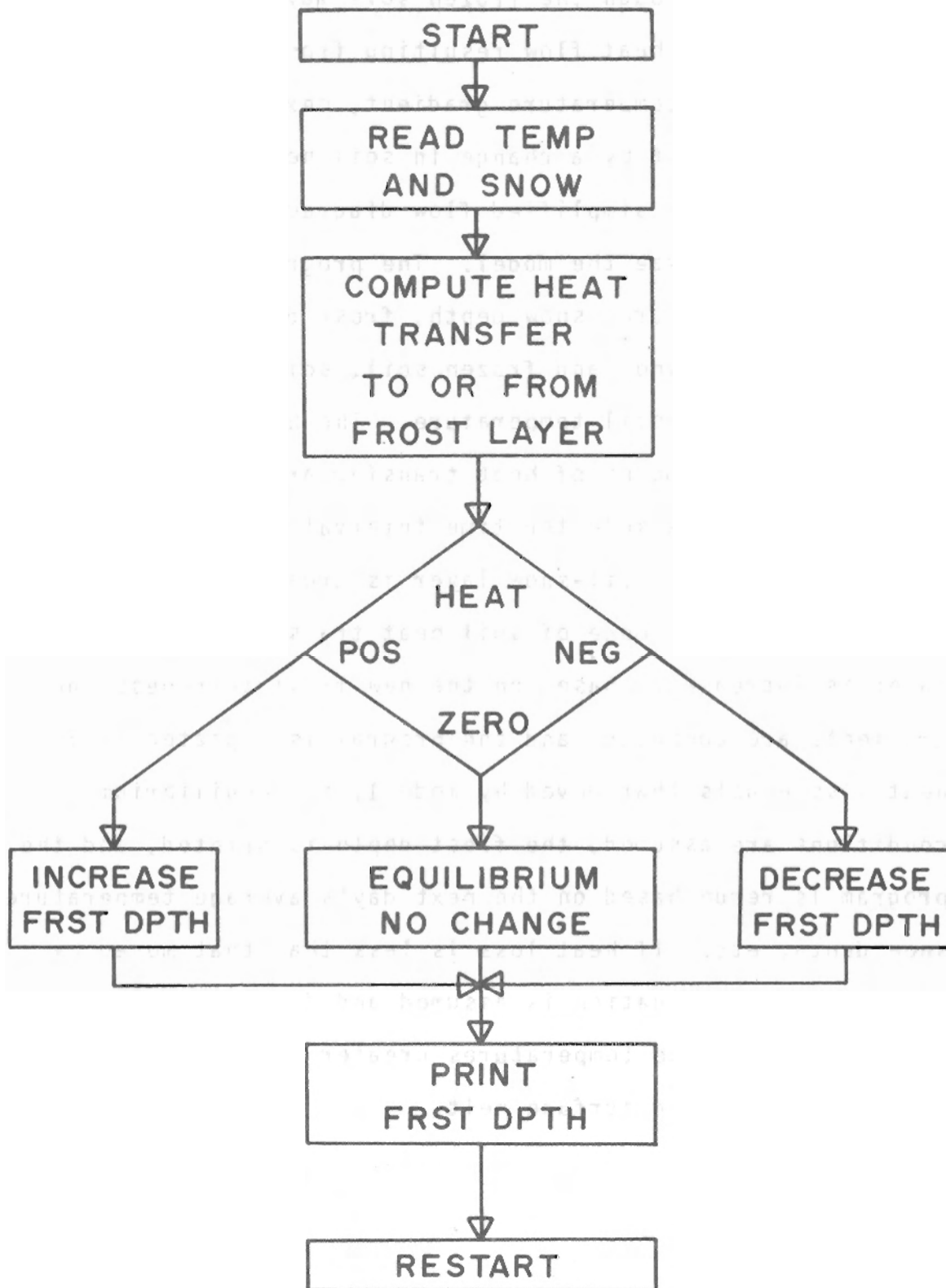
$$\bar{K} = \frac{K_{\text{snow}} K_f}{K_{\text{snow}} x_f + K_f x_{\text{snow}}} (x_f + x_{\text{snow}}) \quad (4)$$

\bar{K} reduces to K_{snow} or K_f if frost depth or snow depth reduces to zero. As the model is analyzed \bar{K} replaces K_F of eq (3).

In developing a program for the model, it was assumed that heat flux through the frozen soil layer was first balanced by heat flow resulting from the soils thermal conductivity and temperature gradient, next by latent heat of fusion, and last by a change in soil heat content ($C_s \Delta v$).

Figure 2 is a simplified flow diagram of the program developed to analyze the model. The program reads the average daily air temperature, snow depth, frost depth, thermal conductivity for snow and frozen soil, soil capillary conductivity, and soil temperature. The heat flow associated with the various modes of heat transfer are then computed and compared for a selected time interval. If the heat lost through the frozen soil-snow layer is greater than that moved by the first mode of soil heat transfer, the frost layer is increased. Based on the new frost thickness, new gradients are computed, and the program is repeated. If heat loss equals that moved by mode 1, and equilibrium conditions are assumed, the frost depth is printed, and the program is rerun based on the next day's average temperature, snow depth, etc. If heat loss is less than that moved by mode 1, a melt situation is assumed and frost depth is decreased. Surface temperatures greater than 0°C are programmed to yield surface melt.

Figure 2. Simplified flow diagram showing major steps and routes considered in frost model.



For example, if we assume a K_F value of 0.3 cal. per cm. min. degree, an average air temperature of -10°C , and a frost depth of 10 cm, equation (1) yields a total heat flow of 0.3 cal for 1 minute of time. Over the same interval of time with an assumed K_S of 0.09, and an unfrozen soil temperature gradient of $0.1^{\circ}\text{C}/\text{cm}$, the first mode of unfrozen soil heat transfer in equation (2) would account for 0.009 cal. Correspondingly, an assumed K_W of 0.008 with unit gradient indicates a potential water transfer of 0.008 cm^3 of water which when frozen would yield a potential total heat transfer by water migration and heat of fusion of 0.64 cal. The difference between heat lost through K_F and heat moved by K_S is 0.291 cal. Since this difference is less than the potential transfer of heat through water migration, water migration presumably will account for the total difference which could require the transfer of $0.291/80 = 0.0035$ cm of water per unit area. With a coefficient of expansion of 1.1, freezing 0.0035 cm of water would cause an increase in frost thickness of 0.0038 cm. Heat transfers are repeatedly recalculated using the new calculated frost thickness each time, until heat transfer by K_F equals K_S . At this point equilibrium is assumed, and predicted frost depth is printed. If equilibrium is not reached in 24 hours, the next day's average temperature and snow depth are read in and used in calculation.

RESULTS

Under bare ground conditions such as found for roads, parking lots, and extremely open winters, frost depth is a function of average air temperature and consequently should also be a function of elevation. Model predictions for bare ground (Figure 3) indicate that time of first frost is earlier, frost depth increases, and frost duration increases as elevation increases. The average air temperatures used for predictions presented here and in future figures were those recorded at East Franklin, Vermont for the 1968-69 winter starting with November 1 as day 1 (Table 1).

Snow acts as an insulator to modify the rate of soil heat loss. Figure 4 presents the effect of snow on predicted frost depth. The model predicts shallower frost penetration with snow cover followed by gradual melting of the frost with time, as snow depth increases. An extension of snow cover effects (Fig. 5) indicates that increased snow depths at higher elevations decrease the presence of frost (points below the 0 line in Fig. 5) at higher elevations despite lower average air temperatures.

The relative proportion of soil heat transferred by the three modes of soil heat flow, proposed in this model, varies as a function of soil water content. This means that as soil water content changes, the speed of frost penetration, depth of penetration and subsequent rate of

Table 1. Average Daily Air Temperature ($^{\circ}\text{C}$) for Winter of 1968-69.

| Day of Month | Nov. | Dec. | Jan. | Feb. | March | April | May |
|--------------|------|-------|-------|-------|-------|-------|------|
| 1 | ---- | - 7.5 | - 6.9 | - 4.1 | - 7.2 | - 7.7 | 4.7 |
| 2 | 1.3 | - 3.6 | -16.3 | - 3.0 | - 6.9 | - 2.7 | 3.6 |
| 3 | 9.1 | 0.0 | -15.8 | - 0.8 | - 1.1 | - 6.9 | 13.8 |
| 4 | 2.7 | - 0.5 | -13.8 | - 7.7 | - 6.9 | - 4.4 | 8.0 |
| 5 | 3.6 | 1.3 | -13.6 | -12.2 | -14.1 | 5.8 | 5.8 |
| 6 | 3.3 | - 1.1 | -10.5 | -16.6 | -14.1 | 2.2 | 4.4 |
| 7 | 1.1 | - 4.4 | - 6.9 | -13.0 | -16.6 | - 0.5 | 7.5 |
| 8 | 2.2 | - 5.5 | - 6.1 | -12.7 | -11.3 | 1.6 | 8.6 |
| 9 | -0.2 | -12.5 | -15.8 | - 8.0 | - 9.1 | - 1.3 | 13.0 |
| 10 | 0.5 | - 3.0 | -11.9 | - 6.6 | - 2.2 | 11.3 | 12.2 |
| 11 | -0.2 | -20.2 | -15.5 | -13.8 | - 7.7 | 6.3 | 3.8 |
| 12 | 0.0 | -10.2 | -15.8 | - 7.2 | -11.1 | - 0.5 | 7.7 |
| 13 | -1.9 | 2.7 | -10.2 | - 1.1 | - 9.4 | 2.5 | 8.3 |
| 14 | -3.0 | 4.1 | - 8.3 | -10.5 | - 5.0 | 8.0 | 5.0 |
| 15 | -4.4 | - 5.5 | - 8.6 | -12.7 | - 1.9 | 10.2 | 5.5 |
| 16 | -1.9 | -14.4 | -15.8 | -12.2 | - 2.2 | 15.5 | 7.7 |
| 17 | -5.2 | -14.7 | -10.5 | -13.6 | - 2.7 | 13.3 | 16.3 |
| 18 | -2.7 | -12.7 | - 1.1 | - 8.0 | - 1.9 | 8.3 | 13.8 |
| 19 | 1.6 | -10.2 | 1.3 | - 6.6 | 2.5 | 6.3 | 10.0 |
| 20 | -0.8 | - 9.1 | - 7.7 | - 3.8 | 3.0 | - 3.6 | 14.4 |
| 21 | -4.1 | - 3.3 | - 7.2 | - 3.8 | 3.0 | 0.5 | 8.3 |
| 22 | -4.1 | - 4.1 | - 2.7 | - 6.6 | 1.3 | 7.5 | 6.1 |
| 23 | -3.0 | - 2.5 | - 0.2 | - 6.1 | - 3.3 | 6.1 | 11.3 |
| 24 | 1.3 | - 2.7 | - 3.6 | 0.2 | 3.0 | 5.5 | 10.0 |
| 25 | 5.0 | | - 1.9 | 0.5 | 6.1 | 3.6 | 12.5 |
| 26 | -2.7 | -27.7 | -12.2 | - 4.4 | 1.6 | 4.1 | 5.2 |
| 27 | -4.7 | -28.0 | -14.1 | - 5.5 | - 3.3 | 5.8 | 2.2 |
| 28 | -3.3 | -20.0 | -19.4 | - 7.2 | - 8.0 | 11.1 | 13.3 |
| 29 | -1.1 | - 5.2 | -16.1 | | - 1.1 | 3.6 | 16.1 |
| 30 | -5.5 | -14.4 | - 6.6 | | - 2.7 | 3.0 | 15.0 |
| 31 | | -15.8 | - 0.5 | | - 6.3 | | 16.9 |

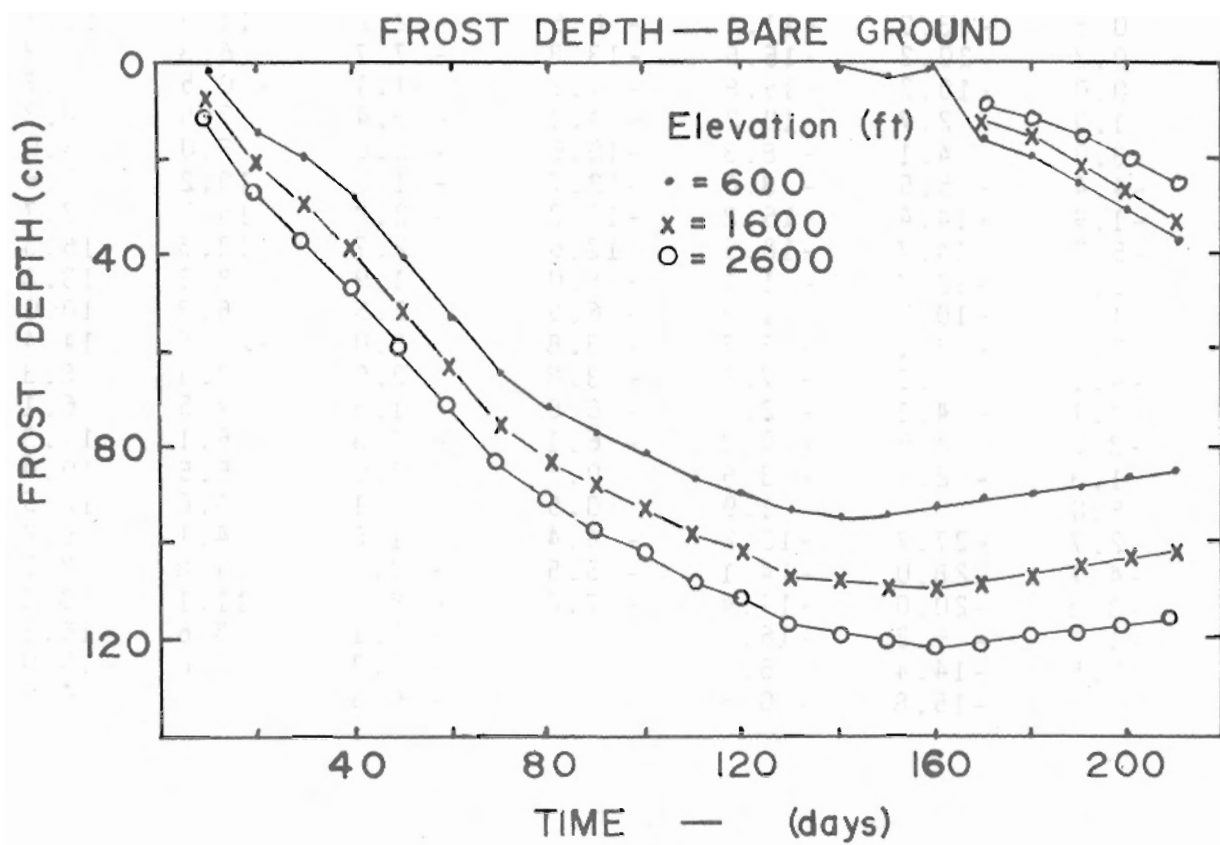


Figure 3. Frost depth under bare ground conditions predicted for decreasing average air temperatures associated with increased elevations.

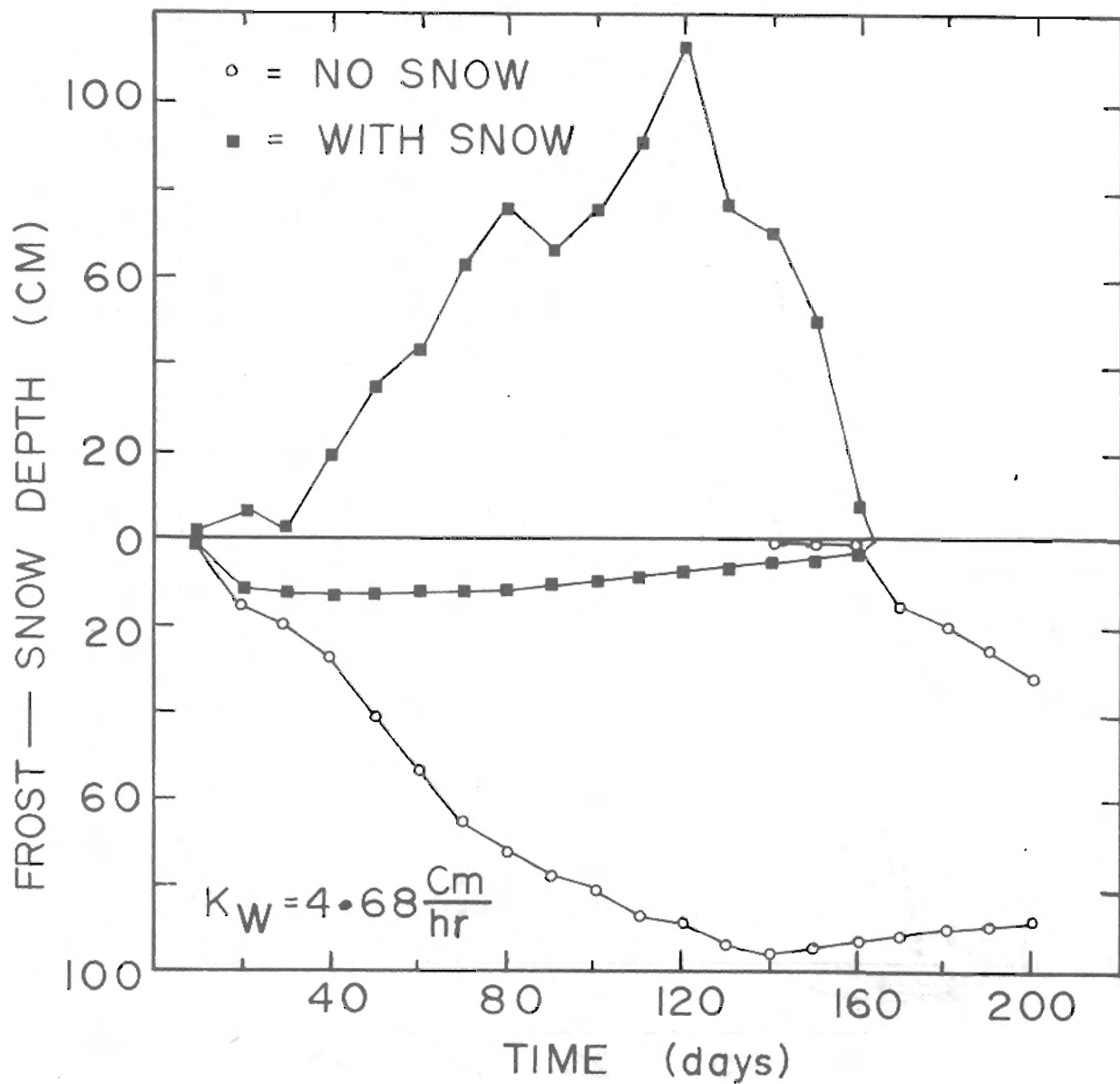


Figure 4. Frost depth for bare ground conditions as compared with frost depth under a snow cover with an assumed capillary conductivity of 4.68 cm/hr.

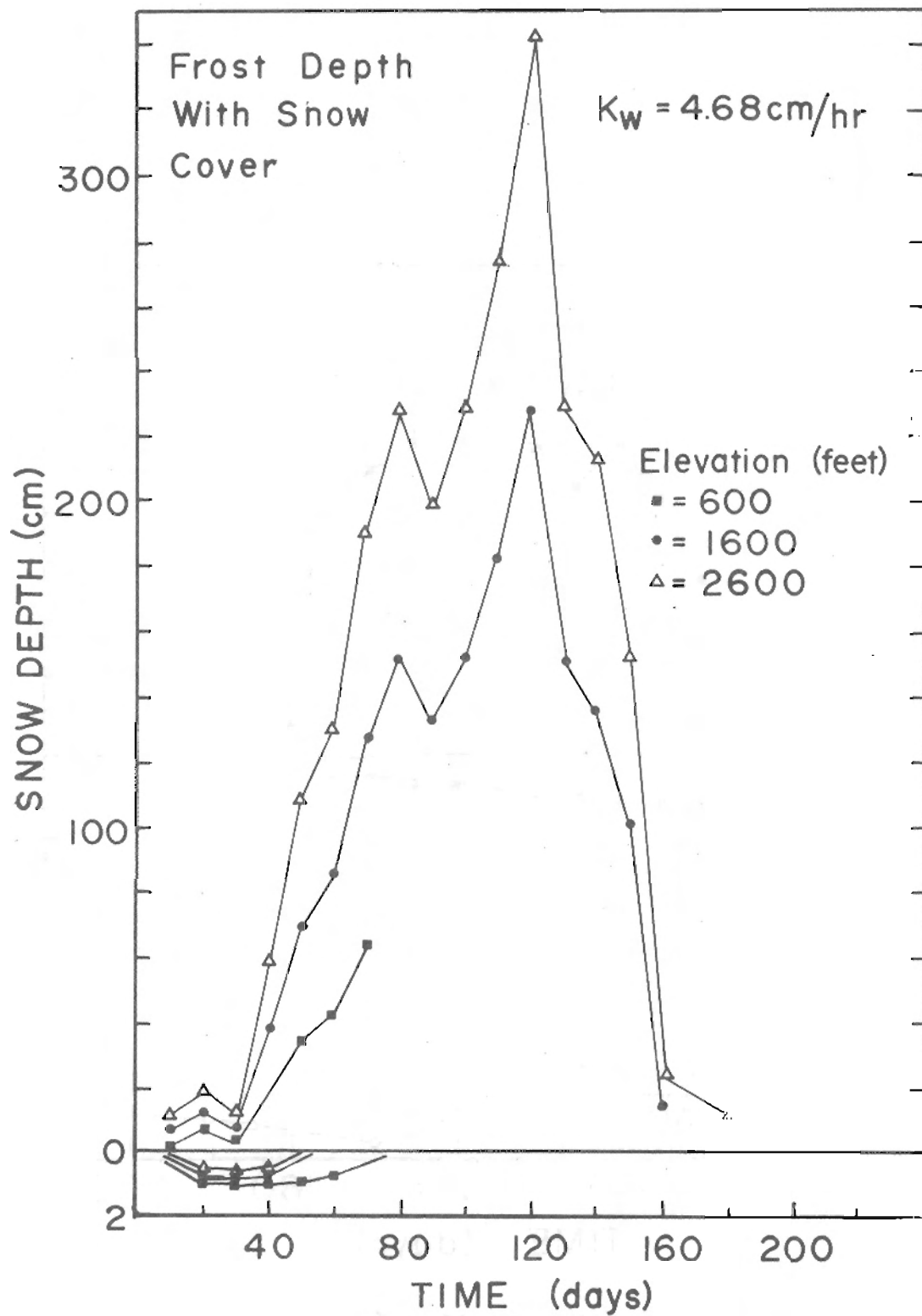


Figure 5. Predicted frost depth associated with increased snow cover at higher elevations for an assumed capillary conductivity of 4.68 cm/hr.

frost melt also change. Figure 6 presents model predictions for frost under three different soil water regimes, as represented by three levels of capillary conductivity for bare ground and one for snow cover. Indications are that soil freezes deeper under dry conditions.

Under normal wet conditions, the model predicts that for a constant freezing temperature near 0°C frost builds up at a rate dependent on the rate of water migration to a freezing plane. Thus, for each cm of water that migrates to the freezing plane, the frost thickness increases by about 1.1 cm. Increased frost thickness results in a smaller temperature gradient, a slower rate of heat loss, and, thus, a slower rate of water migration. The net result is an asymptotic approach to an equilibrium frost thickness when heat loss through the frozen layer equals the soil heat that flows as a function of the soil's temperature gradient. At this point water migration stops.

If the freezing temperature is drastically lowered during the freezing process, the third mode of heat flow ($C_s \Delta v$) operates and results in rapid frost penetration in excess of that associated with the expansion of freezing water. The data presented in Figure 7 verify these predictions which show water uptake with time for shallow soil models frozen while maintained at a constant water tension of either 15 or 30 cm. The initial freezing

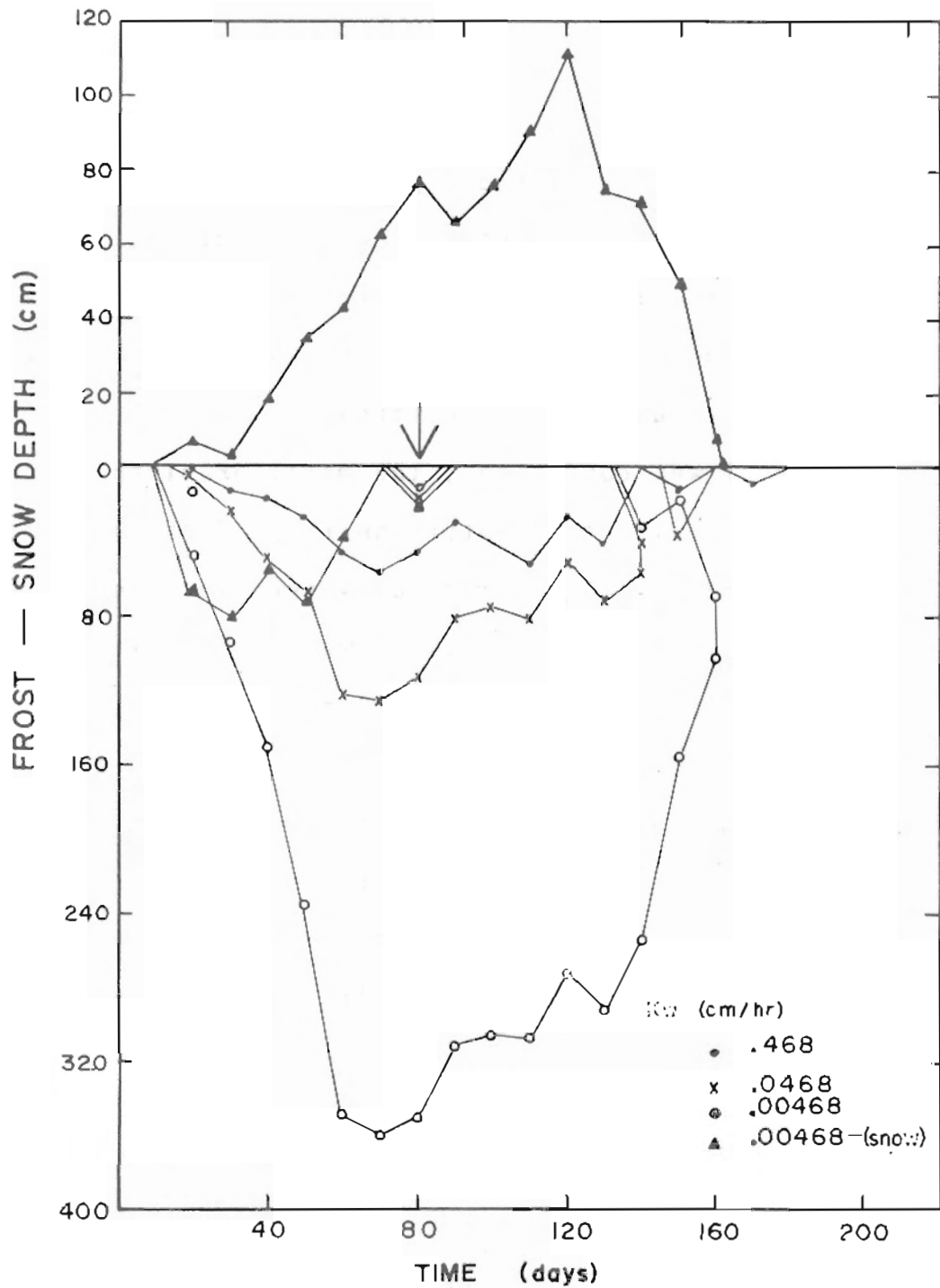


Figure 6. Predicted frost depth variations for three cases under bare ground conditions with different assumed capillary conductivities and one case with snow cover. The model operations for this figure include a modified melt routine as compared with Figures 3 and 4.

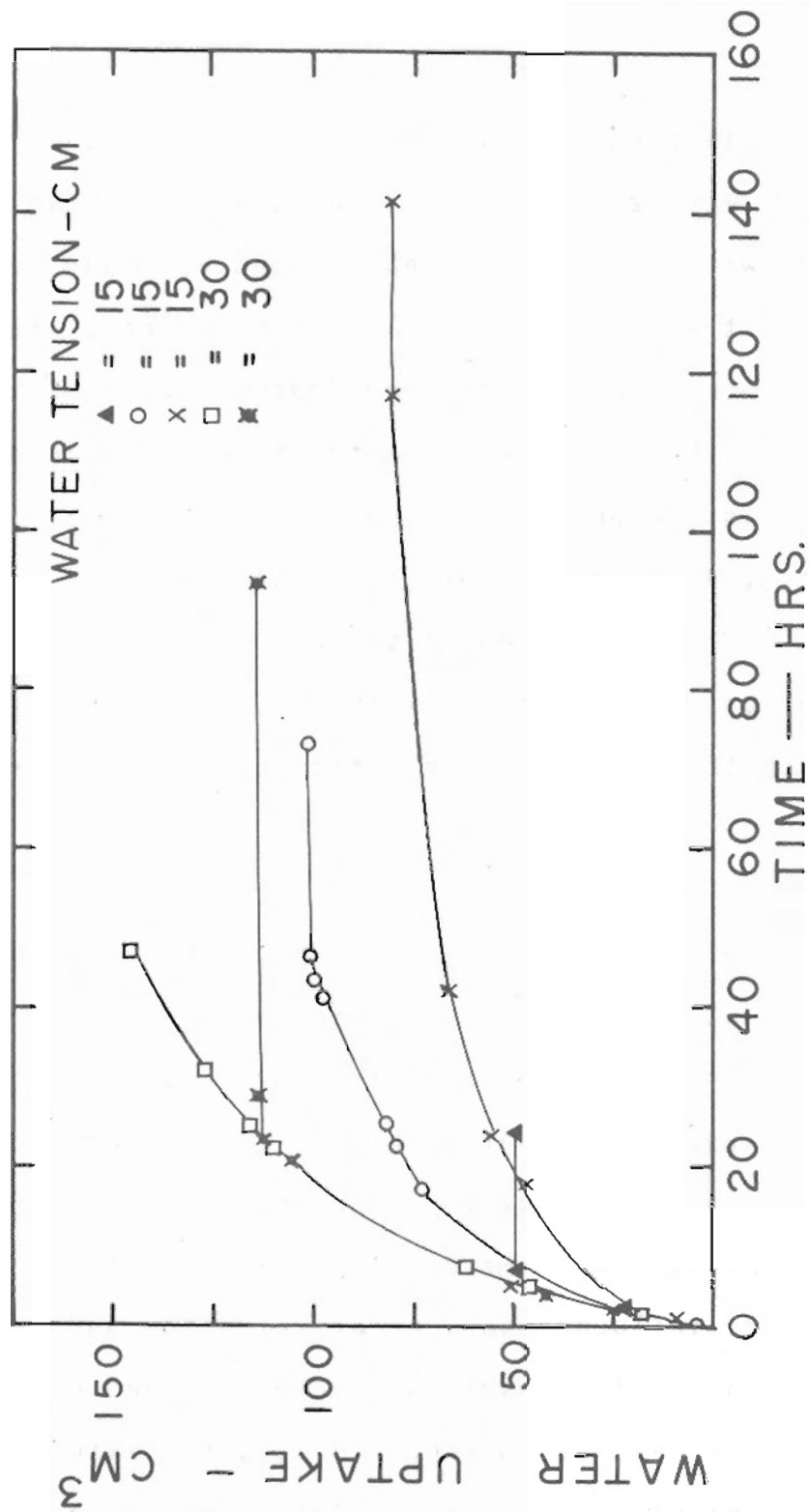


Figure 7. Water uptake during freezing laboratory soil models held at water tension levels of 15 and 30 cm.

temperature imposed on all five soil models was -3°C . This temperature was maintained for the duration of the study for models with data points represented by \square and x . These curves depict a decreasing rate of water uptake with time that corresponded with increased frozen layer thickness. In contrast, the curves represented by \triangle , o , and \times had their freezing temperatures changed to -8°C , -11°C , and -9.5°C at 8, 45, and 24 hours, respectively. At that time the third mode of soil heat loss ($C_s v$) became active causing a rapid penetration of the 0°C isotherm, complete freezing of the model, and cessation of all further water uptake.

DISCUSSION

Soil features that influence the rate and manner of soil heat flow also influence frost depth and distribution. For example, soil texture and structure affect heat flow as a result of associated variations in (1) number of contact points through which heat can flow and (2) path length of heat flow through solid material. More important, texture and structure influence soil water retention and transmission which profoundly affect soil heat capacity and heat flow through latent heat of fusion. Thus, soil texture, structure, and water content are all properties that should affect frost depth and distribution.

Dry soils with a low heat capacity and low rates of water transmission will exhibit deep and rapid frost penetration (Fig. 6) and should have little tendency for heaving. In contrast, wet soils with high rates of water migration are most susceptible to frost heaving with high levels of water accumulation at the freezing plane, but slow penetration of the zero degree isotherm. This behavior is predicted by the model (Fig. 6) and is partially verified by data (Fig. 7).

The model predicts deeper frost penetration with lower temperatures (Fig. 3). This ordinarily would indicate that the lower temperatures at high elevations would result in deeper frost penetration and a harsher soil environment. However, high elevations also experience deeper snowpacks which act to slow the rate of soil heat loss. The model predicts that the net effect for the temperatures (Table 1) and snowpack of the winter of 1968-69 was little if any frost at elevations above 600 ft (Figs. 4 & 5). This indicates that the environment of mountain soils may be less harsh than expected and leads to the speculation that melt water from the bottom of mountain snowpacks may act as the winter water source for northern streams.

In its present stage, the proposed model is a physical conceptual approach to determining frost depth and distribution. The predictions it yields indicate the type of effect various soil features and snow depth may have on frost. Future work must refine and verify model prediction, as well as determine the degree that each soil and surface parameter influences frost development. Speculation about the interactions that may exist between elevation, temperature, snow depth, and frost presence should be tested against reliable field data.

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